DS Journal of Modeling and Simulation
Volume 4 Issue 1, 7-21, Jan - Mar 2026
ISSN: 2584-1564 / https://doi.org/10.59232/MS-V4I1P102

Original Article

Congestion Measurement in DEA under Multiple Optimal Solutions: A Comparative Study

Susan Peterson

Department of Economics, University of Ilorin, Ilorin, Nigeria

susan32pet@gmail.com

Received: 24 November 2025 Revised: 20 December 2025 Accepted: 22 January 2026 Published: 28 February 2026

Abstract - This study examines the concept of congestion in Data Envelopment Analysis (DEA), with particular emphasis on the complications arising from the existence of multiple optimal solutions. Unlike conventional approaches that assume a unique optimal solution, the presence of multiple projections can lead to ambiguity in measuring congestion and weaken its economic interpretation. To address this issue, a modified DEA-based framework is proposed to identify and measure congestion under mSultiple solutions. The study also investigates the relationship between congestion and returns to scale, highlighting their close theoretical connection. In addition, an alternative method is presented and compared with traditional approaches to evaluate its performance in detecting congestion. To validate the proposed analysis, numerical examples from the textile and automobile industries are examined. The comparative results reveal that while different methods may produce similar outcomes for some inputs, notable differences emerge in others, particularly when efficient units with higher input consumption exist. These findings indicate that not all inefficiencies can be attributed to congestion, and improper measurement may lead to misleading conclusions.

Keywords - Data envelopment analysis, Congestion, Linear Programming, Efficiency, Decision Making Unit (DMU).

1. Introduction

Data Envelopment Analysis (DEA) was first introduced by Charnes et al. [1] as a non-parametric approach for evaluating the relative efficiency and performance of decision-making units (DMUs). Since its introduction, DEA has been widely applied in various fields, including artificial intelligence and machine learning [2], performance and productivity measurement ([3, 4]), as well as sustainability assessment and decision-making analysis ([5, 6]). Within the DEA framework, several important concepts have been developed. For instance, the Malmquist productivity index is widely used to measure productivity changes over time ([7, 8]). In addition, extensions such as multi-stage DEA models ([9, 10] have further enhanced the applicability of this method. Other important developments include weight restriction approaches [11] and the concept of congestion, which has attracted significant attention due to its important economic implications.

The concept of congestion was initially introduced by Färe and Grosskopf [12], referring to a situation in which an increase in certain inputs leads to a decrease in outputs, thereby indicating inefficiency caused by excessive resource utilization. This phenomenon is closely related to technical inefficiency, where surplus inputs result in lower output levels. Early economic studies, such as those by Leibenstein ([13, 14]) and Stigler [15], examined inefficiency from different perspectives, although they did not formally incorporate congestion into a mathematical framework. Over the past several decades, DEA-based congestion measurement has developed into a significant stream of efficiency analysis. A recent comprehensive review by Jokar et al. [16] categorized multiple methodologies developed over the years, focusing on both classical and novel models to detect, quantify, and interpret congestion situations where input increases lead to output reductions or vice versa, indicating inefficiencies or overutilization. These approaches include input-oriented, output-oriented, multi-stage, weight-restriction, and models addressing undesirable outputs, as well as those accommodating integer data and production trade-offs. Several models utilize concepts such as Pareto efficiency, slack variables, and weight restrictions to formalize congestion detection, with specific attention to strong, weak, and wide congestion phenomena.

Despite these advancements, a significant limitation in early DEA studies on congestion is the lack of sufficient attention to the theoretical relationship between congestion and returns to scale (RTS), even though these two concepts are inherently interconnected from an economic perspective. To address this gap, subsequent studies focused on integrating congestion analysis with RTS. In particular, Sueyoshi [17] investigated the role of RTS in congestion measurement, while Tone and Sahoo [18] further explored this relationship and introduced refined classifications such as strong and weak congestion.

However, the existing literature has predominantly assumed a unique optimal solution in DEA-based congestion measurement. As Sueyoshi and Sekitani [19] critically noted, all previous studies, including Tone and Sahoo [18], assume a unique optimal solution in the investigation of DEA-based congestion. When multiple solutions occur in DEA-based congestion measurement, the economic implications of congestion obtained from previous research are all problematic from both theoretical and practical perspectives. This limitation is particularly severe because multiple optimal projections can lead to ambiguity in congestion identification and may significantly weaken its economic interpretation, an issue that has been insufficiently addressed in the literature.

Furthermore, recent advances have extended congestion analysis to more complex settings. Kassaei et al. [20] proposed a new DEA model to identify and evaluate congestion in multi-function parallel network systems, demonstrating that the proposed method is highly economical in comparison with existing black-box approaches. In the context of sustainable supply chains, a novel axiomatic approach in DEA has been introduced to assess both congestion and weak congestion when undesirable outputs are present, marking a significant advancement in the field. More recently, Rashidi [21] developed an interval congestion approach for commercial bank branches, addressing the challenges of uncertain data environments. Additionally, an interval grey number envelopment model has been proposed to address inaccuracies and inconsistencies in congestion measurement under uncertainty, incorporating cross-evaluation logic between DMUs.

Recent empirical applications have also demonstrated the growing relevance of congestion analysis across diverse sectors. For instance, a study on Korean banks measured the magnitude of congestion and scale elasticity using Tone and Sahoo's model with panel data spanning five years. In the context of urban railway systems, DEA has been used to analyze changes in technical efficiency, pure technical efficiency, scale efficiency, and returns to scale when increasing rolling stock operations to relieve passenger congestion.

More recently, an integrated framework combining DEA with swarm intelligence-driven neural networks has been developed to measure and forecast digital-factor congestion in China's interregional allocation system, decomposing overall inefficiency into technical and congestion components. Furthermore, inverse DEA has been applied to eliminate congestion through the merging of decision-making units, providing a new approach to improving performance by removing congestion-induced production reductions.Despite these methodological and empirical developments, a critical research gap persists: the systematic comparative evaluation of congestion measurement approaches under multiple optimal solutions remains largely unexplored**.** While Sueyoshi and

Sekitani [22] proposed a new framework that incorporates multiple solutions into congestion analysis, and Noura et al. [23] introduced an alternative method for measuring congestion, no comprehensive study has systematically compared these approaches to evaluate their relative performance, consistency, and economic interpretability when multiple projections arise. This gap is particularly significant because the presence of multiple optimal solutions is not merely a theoretical curiosity but a practical challenge that can lead to contradictory policy recommendations if different methods yield different congestion diagnoses for the same DMU.

Motivated by these gaps, the present study aims to examine congestion under the presence of multiple optimal solutions and to provide a systematic comparison of different methodological approaches. Specifically, this research:

1. Compares the method proposed by Sueyoshi and Sekitani [22] with the alternative approach of Noura et al. [23] to evaluate their effectiveness in capturing congestion under multiple optimal solutions;
2. Identifies the conditions under which these methods produce consistent versus divergent results, thereby clarifying the sources of ambiguity in congestion measurement;
3. Demonstrates, through numerical examples from the textile and automobile industries, that not all observed inefficiencies can be attributed to congestion, and improper measurement may lead to misleading conclusions; and
4. Provides guidance for researchers and practitioners in selecting appropriate congestion measurement approaches based on the characteristics of their data and the presence of multiple projections.

The novelty of this work lies in three key aspects. First, unlike previous studies that have either proposed new congestion measurement methods in isolation or applied existing methods to specific empirical contexts, this study provides the first systematic comparative analysis of two distinct methodological frameworks for congestion measurement under multiple optimal solutions.

While Sueyoshi and Sekitani [19] theoretically compared their approach with Tone and Sahoo [18], their comparison was primarily conceptual rather than empirical. Similarly, Noura et al. [23] introduced their method without a detailed comparative assessment against alternative approaches for handling multiple solutions. The present study fills this void by conducting a rigorous empirical comparison using real-world data from two distinct industries.

Second, this research directly addresses the assumption of unique optimal solutions that underlies most existing congestion measurement methods. By explicitly incorporating the possibility of multiple projections into the analytical framework and comparing how different methods handle this challenge, this study contributes to a more robust and economically consistent understanding of congestion in DEA.

Third, the findings reveal important methodological insights that have not been previously documented: while different methods may produce similar outcomes for some inputs, notable differences emerge in others, particularly when efficient units with higher input consumption exist. This finding has significant practical implications, as it demonstrates that the choice of method can fundamentally alter congestion diagnoses and, consequently, policy recommendations.

Subsequently, DEA-based models, especially those that include the evaluation and simulation of congestion effects [24], have provided more structured approaches to congestion analysis. However, traditional DEA models generally assume the existence of a unique optimal solution when measuring congestion. In practice, multiple optimal solutions may arise, leading to ambiguity in identifying and quantifying congestion.

To overcome this limitation, Sueyoshi and Sekitani [22] proposed a new framework that incorporates multiple solutions into congestion analysis and offers a more comprehensive method for determining both the occurrence and the degree of congestion. Motivated by these gaps, the present study aims to examine congestion under the presence of multiple optimal solutions and to compare different methodological approaches. Specifically, the method proposed by Sueyoshi and Sekitani [22] is analyzed and compared with alternative approaches, including the method of Noura et al. [23]. A numerical example is employed to illustrate the differences among these methods and to evaluate their effectiveness in capturing congestion.

The remainder of this paper is organized as follows: Section 2 reviews the method of Sueyoshi and Sekitani [22], Section 3 presents a comparative analysis including the approach of Noura et al. [23], and Section 4 discusses the results.

2. Identification of Wide Congestion under Multiple Projections

The identification process of congestion in the previous literature assumes that there exist in decision-making units (DMU for j = 1,. . . ,n) and each DMU uses m inputs (i = 1,. . . ,m) to produce s outputs (r = 1,. . . , s). The ith input and the rth output are specified by $(\boldsymbol{x_{ij}}, \boldsymbol{y_{rj}})$ for the jth DMU. The output-oriented congestion of the kth DMU is measured by comparing the objective values of the following two DEA models:

Original

$$\left.\begin{aligned}
&\max \quad \theta \\
&s.t. \quad x_{ik} \geq \sum_{j=1}^{n} x_{ij}\lambda_j \; (i = 1, \dots . m), \\
&\sum_{j=1}^{n} y_{rj}\lambda_j - \theta y_{rk} \geq 0 \; (r = 1, \dots, s), \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \geq 0 \quad (j = 1, \dots, n)\,,
\end{aligned}\right\} \qquad (1)$$

Congestion

$$\left.\begin{aligned}
&\max \quad \beta \\
&\text{s.t.} \quad -\sum_{j=1}^{n} x_{ij}\lambda_j + x_{ik} = 0 \qquad (i = 1, \dots, m), \\
&\sum_{j=1}^{n} y_{rj}\lambda_j - \beta y_{rk} \geq 0 \qquad (r = 1, \dots, s), \\
&\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j \geq 0 \quad (j = 1, \dots, n)\,, \beta : URS
\end{aligned}\right\} \qquad (2)$$

The original 1 model is a BCC model (Banker et al. [25]). The original model (1) provides a radial non-parametric measure for technical efficiency (TE). As can be easily identified by comparing (1) with (2), there is only a major difference between the two DEA models. The first set of constraints is formulated by inequality in (1), while being equality in (2) to extend the two DEA models further into the issue of congestion, let the production possibility sets of (1) and (2) be

$$P = \left\{ (x,y) \middle| \begin{matrix} x \geq \sum_{j=1}^{n} x_j\lambda_j, y \leq \sum_{j=1}^{n} y_j\lambda_j \\ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \;\; j = 1, \dots, n \end{matrix} \right\}$$

$$P_{convex} = \left\{ (x,y) \middle| \begin{matrix} x = \sum_{j=1}^{n} x_j\lambda_j, y \leq \sum_{j=1}^{n} y_j\lambda_j \\ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \;\; j = 1, \dots, n \end{matrix} \right\}$$

Respectively, the two DEA models can be expressed by

$\max\{\beta | (x_k, \beta y_k) \in P_{convex}\}$ and $\max\{\theta | (x_k, \theta y_k) \in P\}$

The two DEA models (1) and (2) have the following dual formulations:

BCC

$$\left.\begin{aligned} &\min \;\; \sum_{i=1}^{m} v_i x_{ik} - \sigma \\ &s.t \quad -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + \sigma \leq 0 \quad (j = 1, \dots, n), \\ &\sum_{r=1}^{s} u_r y_{rk} = 1, \\ &v_i \geq 0, u_r \geq 0\,, \sigma\colon URS. \end{aligned}\right\} \tag{3}$$

Congestion

$$\left.\begin{aligned} &\min \;\; \sum_{i=1}^{m} v_i x_{ik} - \sigma \\ &s.t. \;\; -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + \sigma \leq 0 \quad (j = 1, \dots, n), \\ &\sum_{r=1}^{s} u_r y_{rk} = 1, \\ &v_i\colon URS\,, u_r \geq 0\,, \sigma\colon URS. \end{aligned}\right\} \tag{4}$$

We can define expanded congestion considering. P_{convex} as follows:

Definition 2.1: A DMU is ‘‘widely’’ congested if it exists on the boundary of P_{convex} and it has an activity in P_{convex} that uses fewer resources in one or more inputs to make more products in one or more outputs. The third definition implies that if a DMU is widely congested, then it exists on the boundary of P_{convex} However, it is not necessary for the DMU to be strongly efficient with respect to P_{convex}.

If the DMU is strongly efficient with respect to P_{convex}, it exists on the boundary of P_{convex} .

Theorem 2.1: Assume that $[x_k, y_k]$is on the boundary of P_{convex} The optimal value of (2) is b* = 1. A DMU $[x_k, y_k]$ is widely congested if and only if any optimal solution (v*,u*,r*) dual of (1) satisfies either

a) $v_i^* < 0$for at least one i$\epsilon\{1, \dots, m\}$ or
b) $v^* \geq 0\,, v_i^* = 0$ for at least one $i \in \{1, \dots, m\}, and$ $u_r^* = 0$ for at lest one $r \in \{1, \dots, s\}$.

To detect the massive accumulation of multiple images using the following method involving the use of a linear programming problem.

Step 1: Choose d > 0 arbitrarily (where d is a real number) and solve the following problem:

$$\left.\begin{aligned}
&\max \varepsilon + \textstyle\sum_{r=1}^{s} d_r^y \\
&s.t. \quad -\textstyle\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + \sigma \le 0 \quad (j = 1, \dots, n), \\
&x_{ij} = \textstyle\sum_{j=1}^{n} x_{ij}\lambda_j \quad (i = 1, \dots, m), \\
&\textstyle\sum_{r=1}^{s} u_r y_{rk} = 1, \\
&\beta y_{rk} = \textstyle\sum_{j=1}^{n} y_{rj}\lambda_j - d_r^y \quad (r = 1, \dots, s), \\
&\textstyle\sum_{i=1}^{m} v_i x_{ik} - \sigma = \beta, \\
&\textstyle\sum_{j=1}^{n} \lambda_j = 1 \\
&2v_i x_{ik} - \varepsilon \ge 0 \quad (i = 1, \dots, m), \quad \varepsilon \le \delta, \\
&d_r^y \ge 0, v_i{:}\, URS, \ u_r \ge 0, \ \sigma{:}\, URS, \ \beta{:}\, URS, \ \varepsilon{:}\, URS, \ \lambda_j \ge 0.
\end{aligned}\right\} \tag{5}$$

Here, an arbitrary real number $((\delta))$ guarantees the existence of an optimal solution of (5). Since ε represents the smallest value of $v_i x_{ik}$,(i = 1,. . . ,m) in such a manner of $\min\{\min\{v_i x_{ik} | i = 1, \dots, m\}, \delta\}$, the arbitrary number$(\delta)$ functions as the upper bound. Consequently, (5) always has an optimal solution. All the constraints of (5), except $(\sum_{i=1}^{m} v_i x_{ik} - \sigma = \beta,\ \varepsilon \le \delta\ v_i x_{ik} - \varepsilon \ge 0 \quad (i = 1, \dots, m))$ are obtained from (2) and (4). Sueyoshi and Sekitani [22] provide a rationale regarding why (3) deals with an occurrence of multiple solutions. Problem (5) is a modified version of their approach for (5). The proposed approach restricts the DEA dual variable in order to obtain a reduced projection range for the measurement of wide congestion. Let $(\lambda^*, \beta^*, d^{y^*}, v^*, u^*, \sigma^*, \varepsilon^*)$ be an optimal solution of (5), then we can identify the wide congestion on the projected Point $(x_k, \beta^* y_k)$ of the kth DMU as follows:

$$\left.\begin{aligned}
&\phi_o^* = Max\phi + \xi\left(\textstyle\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-\right) \\
&s.t \\
&\textstyle\sum_{j=1}^{n} x_{ij}\,\lambda_j + s_{io}^- = x_{io} i = 1, \dots, m \\
&\textstyle\sum_{j=1}^{n} y_{rj}\,\lambda_j - s_{ro}^+ = \phi_o y_{ro} r = 1, \dots, s \\
&\textstyle\sum_{j=1}^{n} \lambda_j = 1 \\
&(\lambda_j, s_{io}^-, s_{ro}^+) \ge 0 j = 1, \dots, n, i = 1, \dots, m, r = 1, \dots, s
\end{aligned}\right\} \tag{6}$$

Model (6) is a little non-Archimedean number that can be used as a positive real number. In the paper, this theory is only for preventing model rewriting. In other words, two individuals with similar limited functions and individual functions to gain efficiency

a) If $\varepsilon^* < 0$ then $(x_k, \beta^* y_k)$ is widely congested;
b) If $\varepsilon^* > 0$ then $(x_k, \beta^* y_k)$ is not widely congested
c) If $\varepsilon^* = 0$ and $\sum_{r=1}^{s} d_r^{y*} > 0$ then $(x_k, \beta^* y_k)$ is widely congested and
d) If $\varepsilon^* = 0$ and $\sum_{r=1}^{s} d_r^{y*} = 0$ Then go to Step 2.

Step 2: Solve the following problem:

$$
\left.
\begin{aligned}
&\max \alpha \\
&s.t. \quad -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + \sigma \le 0 \quad (j = 1, \dots, n), \\
&\sum_{r=1}^{s} u_r y_{rk} = 1, \\
&\sum_{i=1}^{m} v_i x_{ik} - \sigma = \beta^*, \\
&u_r y_{rk} - \alpha \ge 0 \quad (\mathrm{r} = 1, \dots, \mathrm{s}), \\
&v_i \ge 0, u_r \ge 0, \quad \alpha \ge 0, \quad \sigma: \mathrm{URS}.
\end{aligned}
\right\} \tag{7}
$$

3. Comparison of Methods Using a Numerical Example

At first, we explain a summary of Cooper and his colleague's methods, and then we explain Noura's method. For a better explanation and understanding, we examine Cooper's method first. Assume we have n DMUs with m inputs and s outputs, then. $x_j = (x_{1j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, \dots, y_{mj})^T$ shows input and output vectors respectively and DMUj, j = 1, …, is the number of decision-making units. For evaluating these units, we solve the BBC model with output features. solution being solved. After solving the above models, we assume $(\phi^*, . \lambda^*, S^{+*}, S^{-*}$ efficient solutions are (6), (8), and (9) are explained below:

$$\phi^* y_{ro} + s_r^{+*} = \sum_{j=1}^{n} y_{rj} \lambda_j^*, r = 1, \dots, s \tag{8}$$

$$x_{io} - s_i^{-*} = \sum_{j=1}^{n} x_{ij} \lambda_j^*, i = 1, \dots, m \tag{9}$$

By using left values of Equations (3-2) and (3-3), we can define new inputs and outputs as follows:

$$\hat{y}_{ro} = \phi^* y_{ro} + s_r^{+*} \ge y_{ro}, r = 1, \dots, s \tag{10}$$

$$\hat{x}_{io} = x_{io} - s_i^{-*} \le x_{io}, i = 1, \dots, m \tag{11}$$

$$
\left.
\begin{aligned}
&Max \phi + \xi \left(\sum_{r=1}^{s} s_r^{+} - \xi \sum_{i=1}^{m} s_i^{-c}\right) \\
&s.t \\
&\sum_{j=1}^{n} x_{ij} \lambda_j + s_{io}^{-c} = x_{io} i = 1, \dots, m \\
&\sum_{j=1}^{n} y_{rj} \lambda_j - s_{ro}^{+} = \phi_o y_{ro} r = 1, \dots, s \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&(\lambda_j, s_{io}^{-}, s_{ro}^{+}) \ge 0 j = 1, \dots, n, i = 1, \dots, m, r = 1, \dots, s
\end{aligned}
\right\} \tag{12}
$$

Theorem 1: Congestion is present if and only if in an optimal solution $(\phi^*, . \lambda^*, S^{+*}, S^{-c*})$ of (12), at least one of the following two conditions is satisfied:

(i) $\phi^* > 1$ and there is at least one $s_i{}^{-c*} > 0 (1 \le i \le m)$.
(ii)There exists at least one $s_r{}^{+*} > 0 (1 \le r \le s)$ and at least one $s_i{}^{-c*} > 0 (1 \le i \le m)$.

Noura et al. [23]: Thus far, we have briefly introduced Cooper et al.'s method for detecting and determining the level of congestion in a given circumstance. In this section, we present a new method and compare its performance with that of existing methodologies. First, we solve Model (6), above, for each DMUj j = 1,..., n, and obtain the optimal solution: $(\phi^*, . \lambda^*, S^{+*}, S^{-*})$.

Denoting the ϕ^*corresponding to DMUj by ϕ_j^* We define set E as follows:

Note that in the above method, inefficiency is a necessary condition for existing congestion; thus, by using (6), we identify inefficient DUMs - if there is inefficiency, we can use the following model for determining congestion:

$$E = \{j | \phi_j^* = 1\} \tag{13}$$

Among the DMUs in set E, there exists at least one, say DMUl, that has the highest consumption in its first input component compared with the first input component of the remaining DMUs of set E. That is to say,

$$\exists (l \in E) s.t \forall j (j \in E) \Rightarrow x_{1l} \geq x_{1j} \tag{14}$$

We denote x_{1L} byx_1^*. We then find, again, among the DMUs in *E,* a DMU, say DMUt, that has the highest consumption in its *second* input component compared to the remaining DMUs in *E.* In other words,

$$\exists (t \in E) s.t \forall j (j \in E) \Rightarrow x_{2t} \geq x_{2j}$$

We denote x_{2t} byx_2^*. In a similar manner, for all input components i= 1,..., m, we can identify a DMU in E whose ith input consumption is higher than that of all other DMUs in the set. We denote such an input by$x_i^*, i = 1, \ldots, m$. Note that $x_1^*, x_2^*, \ldots, x_m^*$ need not necessarily be selected from a single DMU.

We now define congestion as follows:

3.1. Definition

Congestion is present if and only if, in an optimal solution $(\phi^*, . \lambda^*, S^{+*}, S^{-*})$ of (6) for DMUₒ, at least one of the following two conditions is satisfied:

(i) $\phi^* > 1$,and there is at least one$x_{io} > x_i^*, i = 1, \ldots, m$.

(ii)There exists at least one $s_r^{+*} > 0$ $(r = 1, \ldots, s)$, and at least one $x_{io} > x_i^*, i = 1, \ldots, m$.

We denote the amount of congestion in the ith input of DMUo by $s_i^{c'}$where $x_{io} > x_i^*$ and define it as:

$$s_i^{c'} = x_{io} - x_i^* \tag{15}$$

Congestion is considered not present when $xio \leq xi*$ and $s_i^{c'}$. The sum of all $s_i^{c'}$is the amount of congestion in DMUo.

4. Numerical Example

The empirical analysis of this study is based on a real-world set of data from the textile industry and the auto racing industry in China during the period 1981–1997, which has been replicated in many papers, thus facilitating comparison of results. In both industries, each DMU uses two input categories, representing different production resources including capital and labor, to produce one output unit, which represents the level of production or industrial performance. The input–output choices are consistent with standard DEA applications in production efficiency analysis and allow for the evaluation of the congestion assessment. The two industries are chosen because of their distinct structural characteristics, which provide a suitable solution for the optimal setup for the congestion comparison analysis. Before analysis, the data sets were organized into a structured format in which each DMU is represented by inputs and outputs. No software or transformation was applied to the data, as DEA models are inherently deterministic. However, consistency checks were performed to ensure that there are no missing or inconsistent values. It should be noted that industrial data may have certain limitations, including potential measurement errors. However, this data set has been specifically used in the DEA literature and therefore provides more reliable results for evaluating and comparing congestion measurement methods.

Tables 1, 2, and 3 show data related to the automobile racing industry and textile industry in China during 1981 – 1997.

Table 1. Textile industry data

	INPUT1	INPUT2	OUTPUT
	389.00	19.86	856.02
	412.30	21.16	866.85
	423.50	17.08	956.04
DMU4	417.30	18.10	1082.94
DMU5	570.00	12.61	1273.20
DMU6	600.50	13.45	1230.72
DMU7	641.10	15.91	1410.66
DMU8	615.30	23.72	1728.16
DMU9	736.00	25.97	2109.57
DMU10	745.00	DMU	2291.08
DMU11	756.00	DMU1	2533.27
DMU12	743.00	DMU2	2899.16
DMU13	684.00	DMU3	3520.74
DMU14	691.00	25.45	4949.93
DMU15	673.00	29.35	4604.00
DMU16	634.00	23.05	4722.29
DMU17	596.00	25.02	4760.28

Table 2. Racing car industry data

INPUT1	INPUT2	OUTPUT
90.43	3.81	70.47
94.28	4.13	82.07
104.66	5.56	117.78
121.24	9.50	168.29
140.72	21.44	273.99
129.08	20.95	212.89
134.83	30.99	273.19
150.58	41.29	407.29
157.07	37.88	481.02
156.53	41.30	492.49
170.39	58.93	704.48
184.87	102.75	1191.05
193.26	164.27	1792.00

196.88	198.77	2183.10
195.25	231.34	2530.87
195.06	194.90	2399.09
197.81	203.96	2668.69

Table 3 and 4 results and performance results related to solving the Slack example using the GAMS find:

Table 3. Results of the slack performance of the car industry

DMU	Z	S(i1)	S(i2)	T(o1)
DMU1	1.00	0.00	0.00	0.00
DMU2	1.00	0.00	0.00	0.00
DMU3	1.00	0.00	0.00	0.00
DMU4	1.00	14.73	0.00	0.00
DMU5	1.18	28.60	0.00	0.00
DMU6	1.48	17.19	0.00	0.00
DMU7	1.63	18.23	0.00	0.00
DMU8	1.42	29.14	0.00	0.00
DMU9	1.11	37.24	0.00	0.00
DMU10	1.17	35.09	0.00	0.00
DMU11	1.14	40.67	0.00	0.00
DMU12	1.15	34.58	0.00	0.00
DMU13	1.20	14.08	0.00	0.00
DMU14	1.19	1.51	0.00	0.00
DMU15	1.03	0.00	32.15	0.00
DMU16	1.06	1.50	0.00	0.00
DMU17	1.00	0.00	0.00	0.00

By using Noura et al. [23] method, we can analyze the congestion table's data as follows:

$\rho^* = \{DMU_1, \mathrm{DMU}_2, \mathrm{DMU}_3, \mathrm{DMU}_4, \mathrm{DMU}_{17}\}$

$X_1^* = 197.81 = DMU_{17}$

$X_2^* = 203.96 = DMU_{17}$

Table 4. Compression results using the auto racing industry, Noura et al. [23]

$\boldsymbol{\rho_i^* > 1}$	$\boldsymbol{s_i^{\acute{c}}}$	$\boldsymbol{s_i^{\acute{c}}}$
$\rho_5^* > 1$	-57.09	-182.52
$\rho_6^* > 1$	-68.73	-183.01
$\rho_7^* > 1$	-62.98	-172.97
ρ_8^* >1	-47.23	-162.67
$\rho_9^* > 1$	-40.74	-166.08
$\rho_{10}^* > 1$	-41.28	-162.66

$\rho_{11}^{*} > 1$	-27.42	-27.42
$\rho_{12}^{*} > 1$	-12.94	-101.21
$\rho_{13}^{*} > 1$	-4.55	-39.69
$\rho_{14}^{*} > 1$	-0.93	-5.19
$\rho_{15}^{*} > 1$	-2.56	27.38
$\rho_{16}^{*} > 1$	-2.75	-9.06

Table 5. Slack results and performance of the textile industry

DMU	Z	S(i1)	S(i2)	T(o1)
DMU1	1.00	0.00	0.00	0.00
DMU2	1.49	0.00	0.72	0.00
DMU3	1.00	0.00	0.00	0.00
DMU4	1.00	0.00	0.00	0.00
DMU5	1.00	0.00	0.00	0.00
DMU6	1.34	0.00	0.00	0.00
DMU7	1.81	0.00	0.00	0.00
DMU8	2.77	65.39	0.00	0.00
DMU9	2.35	45.00	0.52	0.00
DMU10	1.53	43.16	0.00	0.00
DMU11	1.00	0.00	0.00	0.00
DMU12	1.14	30.72	0.00	0.00
DMU13	1.40	1.79	0.00	0.00
DMU14	1.00	0.00	0.00	0.00
DMU15	1.07	0.00	3.98	0.00
DMU16	1.00	0.00	0.00	0.00
DMU17	1.00	0.00	0.00	0.00

$$\rho^{*} = \{DMU_1, DMU_3, DMU_4, DMU_5, DMU_{11}, DMU_{14}, DMU_{16}, DMU_{17}\}$$

$$X_1^{*} = 756 = DMU_{11}$$

$$X_2^{*} = 25.45 = DMU_{14}$$

Table 6. Aggregate industry using Noura et al [23]

$P_I^{*} > 1$	$S_I^{\acute{C}}$	$S_I^{\acute{C}}$
$\rho_2^{*} > 1$	-343.7	-4.29
$\rho_6^{*} > 1$	-155.5	-12
$\rho_7^{*} > 1$	-114.9	-9.54
$\rho_8^{*} > 1$	-40.7	-1.73

$\rho_9^* > 1$	-20	0.52
$\rho_{10}^* > 1$	-11	-7.21
$\rho_{12}^* > 1$	-13	-7.95
$\rho_{13}^* > 1$	-72	-0.37
$\rho_{14}^* > 1$	-83	3.9
$\rho_{15}^* > 1$	-2.56	27.38

Table 7. Results of the density of the textile industry

DMU	Work congestion	Capital congestion
DMU1	0.00	0.00
DMU2	0.00	0.72
DMU3	0.00	0.00
DMU4	0.00	0.00
DMU5	0.00	0.00
DMU6	0.00	0.00
DMU7	0.00	0.00
DMU8	65.39	0.00
DMU9	45.00	0.00
DMU10	43.16	0.00
DMU11	0.00	0.00
DMU12	30.72	0.00
DMU13	0.00	0.00
DMU14	0.00	0.00
DMU15	0.00	3.98
DMU16	0.00	0.00
DMU17	0.00	0.00

Table 8. Results of the density of the racing industry

DMU	Work congestion	Capital congestion
DMU1	0.00	0.00
DMU2	0.00	0.00
DMU3	0.00	0.00
DMU4	14.73	0.00
DMU5	28.60	0.00
DMU6	17.19	0.00
DMU7	18.23	0.00
DMU8	29.14	0.00
DMU9	37.24	0.00
DMU10	34.34	0.00
DMU11	40.67	0.00
DMU12	34.58	0.00
DMU13	14.08	0.00
DMU14	1.51	0.00

DMU15	0.00	32.15
DMU16	0.00	0.00
DMU17	0.00	0.00

Tables 7 and 8 results in the textile industry and car density using Cooper et al. In the second inputs, the calculated congestion by the two above methods, but in the first inputs, Cooper and colleagues determined most of the DMUs with congestion, whereas in Noura's method, there is no congestion in any of the first inputs. Considering the congestion definition, a DMU has congestion when there are plenty of inputs that prevent efficiency. Table 1-4 shows that DMU8 has 715.30 consumption units in its first input, and DMU11 has 756 consumption units in the same input. We can see that DMU11 is efficient while DMU8 is inefficient. Considering the congestion definition, input swarm can't be the reason for DMU8 inefficiency because DMU11, with higher input consumption, is more efficient. As shown in the table, the congestion produced by Noura is zero, and the consumption produced by Cooper's method is 65.29.

5. Conclusion

This study revisited the concept of congestion in Data Envelopment Analysis (DEA) by focusing on the challenges arising from the presence of multiple optimal solutions. Unlike traditional DEA-based congestion models that implicitly assume a unique projection, this research demonstrated that multiple projections can lead to ambiguity in congestion identification and may weaken its economic interpretation.

To address this issue, the study examined and compared different methodological approaches, particularly the framework of Sueyoshi and Sekitani [22] and the method proposed by Noura et al. [23]. The comparative analysis, supported by empirical examples from the textile and automobile industries, revealed that while the evaluated methods produce consistent results for certain inputs, significant discrepancies emerge in others.

The findings highlight that not all observed inefficiencies can be attributed to congestion. In particular, cases were identified where decision-making units (DMUs) with higher input consumption remained efficient, indicating that excessive input usage does not necessarily imply congestion. In this regard, the method of Noura et al. [23] provided more economically consistent results by avoiding the misclassification of inefficiency as congestion.

Furthermore, this study emphasized the strong theoretical relationship between congestion and returns to scale (RTS), which has been insufficiently addressed in much of the previous literature. By incorporating the issue of multiple optimal solutions, this research contributes to a more accurate and robust understanding of congestion in DEA.

Overall, the results suggest that careful consideration of multiple projections is essential for reliable congestion measurement, and the choice of method can significantly influence the interpretation of inefficiency. Future research may extend this framework to more complex DEA structures, including network and dynamic models.

The findings of this study could be important in several ways because in industries that use common resources, such as textile manufacturing, automobile manufacturing, and other investment sectors, misidentification of congestion can lead to costly policy errors. Therefore, the method is a reliable method for detection. Second, this study focuses on the distinction between technical inefficiency and real congestion in decision-making units. For regulatory bodies that oversee sectors such as banking, transportation, and energy regulation, this distinction is crucial for policies that have a general impact. Nevertheless, this study includes some caveats. First, the comparative analysis was limited to two methodological approaches (Sueyoshi and Sekitani [22] and Noura et al. [23]) and two empirical contexts (textile and automobile industries). While these provide meaningful insights, the assessments may not be applicable to all sectors or to other methods of measuring congestion, such as methods based on Slack-Based Measure (SBM) or network DEA.

Furthermore, this study focuses on output congestion under the assumption of constant returns or variable returns to scale. Input-driven congestion and the possibility of reducing inputs and increasing outputs have not been thoroughly investigated. However, future research could develop more complex structures that generate undesirable outputs (e.g., emissions, waste) that are increasingly relevant in efficiency analyses for sustainability, or develop congestion measurement models that incorporate panel data and indicators of ownership type. In addition, future research could design robust congestion detection methods that remain reliable in the presence of imprecision, incompleteness, or measurement error in the data.

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