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Original Article

On Prognosis of Skeletal Muscle Contraction

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Abstract - In this paper, a model for the analysis of skeletal muscle contraction is presented, and its deformation properties are accounted for. It was also presented an analytical approach for analysis of the considered muscle contraction.

Keywords - Muscle contraction, Process model, Analytical approach for analysis.

1. Introduction

Modeling of muscle contraction is an important component in studying the physiological characteristics of human movement. Knowledge of the informative parameters of muscle's mechanical (elastic-viscous) properties is used in medicine and fine arts when considering the influence of subcutaneous muscles on the shape of the human body. In sports, modeling of human muscle movement helps coaches improve the effectiveness of sports training, and the capabilities of modern computers allow conducting research and introducing corrections into the training methodology directly during its implementation [1-5]. In this paper, a model for the analysis of skeletal muscle contraction, which takes into account its deformation properties. The model makes it possible to take into account the spatial and temporal dependences of the parameters of the considered process. The accounting was not find in literature. An analytical approach was also presented to analyze the muscle contraction considered. The approach makes it possible to take into account the spatial and temporal dependences of the parameters of the considered process. The accounting was not found in the literature.

2. Method of Solution

In this section, a model of skeletal muscle contraction was considered and analyzed. In the framework of the model under consideration, we will assume that the muscle is a locally flat object and has the structure "elastic thread - elastic-viscous substrate": it is a set of parallel threads connected to an elastic-viscous substrate. It was assumed that the effective layer of tissue with depth H is reduced. A linear law of distribution along the coordinate q of the component of the displacement field normal to the muscle surface is adopted.

$$U(y,z,t)=V(z,t)[1+\alpha(y,z,t)y/z], \tag{1}$$

Where $U\ (y,z,t)$ is the normal to the muscle surface component of the displacement vector field; $V\ (z,t)$ is the movement of a fiber point along the Oy axis, spaced from the edge at a distance z; H is the depth of the effective layer of the substrate; y is the coordinate directed from the free surface of the muscle; z is the fiber axis coordinate; α is the empirical parameter that takes into account possible deviations of the system under consideration from ideality. The equation of transverse oscillations of a thread on an elastic-viscous substrate has the following form [7, 8].

$$m\frac{\partial^2 V}{\partial t^2}=\frac{\partial}{\partial z}\left[T\frac{\partial V}{\partial z}\right]-q\,, \tag{2}$$

Where m is the mass of a unit of the thread; $T\,(y,z,t)$ is the thread tension force; $q\,(y,z,t)$ is the distributed shear force from the side of the substrate, directed against the axis y. Force $q\,(y,z,t)$ is determined through the tension in the muscle - the substrate σ, multiplied by the effective width b: $q = \sigma\, b$. As boundary conditions, equation (2) is supplemented by the conditions for fastening the thread

$$V(0,t)=0,\ V(L,t)=0, \tag{3}$$

Where L is the effective thread length. Initial conditions for the function $V\,(z,t)$ could be written as

$$V(z,0)=V_0,\ \left.\frac{\partial V(z,t)}{\partial t}\right|_{t=0}=0\,. \tag{4}$$

Equation (2) was solved with conditions (3, 4) by recently introduced method of functional corrections [7, 8]. In the framework of the approach, the thread tension force $T\,(y,z,t)$ was transformed to the following form

$$T=T_0[1+\varepsilon\cdot g(y,z,t)] \tag{5}$$

Where T_0 is the average value of the considered force, $0\le \varepsilon <1$, $|g\ (y,z,t)|\le 1$. We determine the solution of equation (2) as the following power series

$$V=\sum_{i=0}^{\infty}\varepsilon^i V_i\,. \tag{6}$$

Substitution of the considered form of solution (6) and relation (5) into equation (2) and conditions (3, 4), as well as grouping of terms at equal powers of the parameter, ε gives a possibility to obtain equations for functions $V_i\,(z,t)$, boundary and initial conditions for them in the following form

$$m\frac{\partial^2 V_0}{\partial t^2}=T_0\frac{\partial^2 V_0}{\partial^2 z}-q \tag{7}$$

$$m\frac{\partial^2 V_i}{\partial t^2}=T_0\frac{\partial^2 V_i}{\partial^2 z}+T_0\frac{\partial}{\partial z}\left\{g\frac{\partial V_{i-1}}{\partial z}\right\},\ i\ge 1, \tag{8}$$

$$V_i(0,t)=0,\ V_i(L,t)=0,\ \left.\frac{\partial V_i(z,t)}{\partial t}\right|_{t=0}=0,\ i\ge 0;\ V_0(z,0)=V_0,\ V_i(z,0)=0,\ i\ge 1. \tag{9}$$

Equations (7), (8) with conditions (9) were solve by the Fourier variable separation method [9]. The considered solutions could be presented in the following form

$$V_0=\frac{V_0 L}{\pi n}\sum_{n=0}^{\infty}\left[(-1)^n-1\right]\sin\left(\frac{\pi n z}{L}\right)\sin\left(\sqrt{\frac{T_0}{m}}\frac{t}{L}\right)-\frac{T_0}{m}\sum_{n=0}^{\infty}\sin\left(\frac{\pi n z}{L}\right)\sin\left(\sqrt{\frac{T_0}{m}}\frac{t}{L}\right)\int_0^L q\cdot\sin\left(\frac{\pi n z}{L}\right)dz\,, \tag{10}$$

$$V_i=-\frac{\pi n}{L}\frac{T_0}{m}\sum_{n=0}^{\infty}\sin\left(\frac{\pi n z}{L}\right)\sin\left(\sqrt{\frac{T_0}{m}}\frac{t}{L}\right)\int_0^L\left\{g\frac{\partial V_{i-1}}{\partial z}\right\}\cos\left(\frac{\pi n z}{L}\right)dz\,, \tag{11}$$

Spatio-temporal distributions of the movement of a fiber point along the *Oy* axis were analyzed analytically by using the second-order approximation in the framework of the method of function corrections. The approximation is usually enough good approximation for to make qualitative analysis and to obtain some quantitative results. All obtained results have been checked by comparison with the results of numerical simulations.

3. Discussion

In this section, the spatio-temporal distribution of the fiber point displacement along the *Oy* axis is presented. Figure 1 shows typical dependences of the considered distribution on the coordinate during fiber compression for various values of the external force q. An increase in the number of curves corresponds to an increase in the considerate force. Stretching the fiber leads to the opposite result. A similar result was obtained when analyzing the change in fiber over time.

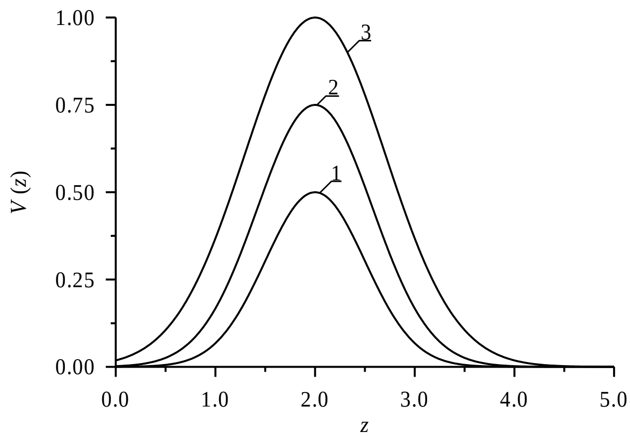

Fig. 1 Typical dependences of the distribution of the fiber point displacement along the *Oy* axis for various values of the external force q. An increase in the number of curves corresponds to an increase in the considerate force

4. Conclusion

In this paper, a model for the analysis of skeletal muscle contraction, which takes into account its deformation properties. The considered model was analyzed. An analytical approach was also introduced to analyze the muscle contraction considered.

Data Availability

All appropriate data are available.

Authors' Contributions

All results of this paper are the author's own.

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