

# On Approach of Analysis of Blood Transport During Cardiac Contractions

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Received: 08 March 2025; Revised: 22 March 2025; Accepted: 02 March 2025; Published: 31 March 2025

**Abstract** - In this paper, we consider a model of blood transport during cardiac contractions. An approach for analyzing the considered model was introduced. The possibility of changing the rate of blood transport is being considered.

**Keywords** - Blood transport, Cardiac contractions, prognosis of the process, Analytical approach for analysis.

## 1. Introduction

The cardiovascular system ensures the transport of blood and all necessary substances to organs and the removal of metabolic products [1-5]. The prognosis of morphological and functional features of the cardiovascular system can prevent or slow down the development of serious diseases that lead to the development of complications in other organs and tissues. Diseases such as arterial hypertension, atherosclerosis, and coronary cardiac disease are widespread and are the most common causes of death due to the development of complications. In this paper, we consider a model of blood redistribution between different reservoirs of the cardiovascular system. We also consider an analytical approach to analyze the above model.

## 2. Method of Solution

In this section, we consider the following model to describe the redistribution of blood between different storage tanks of the cardiovascular system

$$\begin{cases} \frac{dV_i(t)}{dt} = Q_{ki}(t) - Q_{ij}(t) \\ P_i(t) = G_i(t)[V_i(t) - W_i(t)] \\ Q_{ij}(t) = \frac{P_i(t) - P_j(t)}{R_{ij}(t)} \end{cases} \quad (1)$$

Where  $i, j, k$  are the numbers of the considered storage tanks;  $V_i(t)$  is the value of blood in the  $i$ -th storage tank;  $P_i(t)$  is the pressure of the considered in the  $i$ -th storage tank;  $G_i(t)$  is the bulk elasticity  $i$ -th storage tank;  $W_i(t)$  is the unstressed volume  $i$ -th storage tank;  $Q_{ij}(t)$  is the flow of blood from  $i$ -th in  $j$ -th storage tank;  $R_{ij}(t)$  is the resistance to flow of blood from  $i$ -th to  $j$ -th storage tank. The initial condition for the considered volume of blood could be written as  $V_i(0) = V_{i0}$ . The solution of the system of Equation (1) could be written as

$$\begin{aligned}
 V_i(t) &= \left\{ V_{i0} - \int_0^t \frac{G_i(\vartheta)}{R_{ij}(\vartheta)} V_i(\vartheta) \exp \left[ - \int_0^\vartheta Q_{ki}(\tau) d\tau - \int_0^\vartheta \frac{P_j(\tau)}{R_{ij}(\tau)} d\tau - \int_0^\vartheta \frac{G_i(\tau) W_i(\tau)}{R_{ij}(\tau)} d\tau \right] d\vartheta \right\} \times \\
 &\quad \times \exp \left[ \int_0^t Q_{ki}(\tau) d\tau + \int_0^t \frac{P_j(\tau)}{R_{ij}(\tau)} d\tau + \int_0^t \frac{G_i(\tau) W_i(\tau)}{R_{ij}(\tau)} d\tau \right] \\
 P_i(t) &= G_i(t) \left\{ V_{i0} - \int_0^t \frac{G_i(\vartheta)}{R_{ij}(\vartheta)} V_i(\vartheta) \exp \left[ - \int_0^\vartheta Q_{ki}(\tau) d\tau - \int_0^\vartheta \frac{P_j(\tau)}{R_{ij}(\tau)} d\tau - \int_0^\vartheta \frac{G_i(\tau) W_i(\tau)}{R_{ij}(\tau)} d\tau \right] d\vartheta \right\} \times \\
 &\quad \times \exp \left[ \int_0^t Q_{ki}(\tau) d\tau + \int_0^t \frac{P_j(\tau)}{R_{ij}(\tau)} d\tau + \int_0^t \frac{G_i(\tau) W_i(\tau)}{R_{ij}(\tau)} d\tau \right] - G_i(t) W_i(t) \\
 Q_{ij}(t) &= \frac{G_i(t)}{R_{ij}(t)} \left\{ V_{i0} - \int_0^t \frac{G_i(\vartheta)}{R_{ij}(\vartheta)} V_i(\vartheta) \exp \left[ - \int_0^\vartheta Q_{ki}(\tau) d\tau - \int_0^\vartheta \frac{P_j(\tau)}{R_{ij}(\tau)} d\tau - \int_0^\vartheta \frac{G_i(\tau) W_i(\tau)}{R_{ij}(\tau)} d\tau \right] d\vartheta \right\} \times \\
 &\quad \times \exp \left[ \int_0^t Q_{ki}(\tau) d\tau + \int_0^t \frac{P_j(\tau)}{R_{ij}(\tau)} d\tau + \int_0^t \frac{G_i(\tau) W_i(\tau)}{R_{ij}(\tau)} d\tau \right] - \frac{G_i(t) W_i(t)}{R_{ij}(t)} - \frac{P_j(t)}{R_{ij}(t)}.
 \end{aligned} \tag{2}$$

Figures 1 and 2 show typical dependences of the volume of blood in the considered storage tanks on time during filling and emptying of them, respectively.

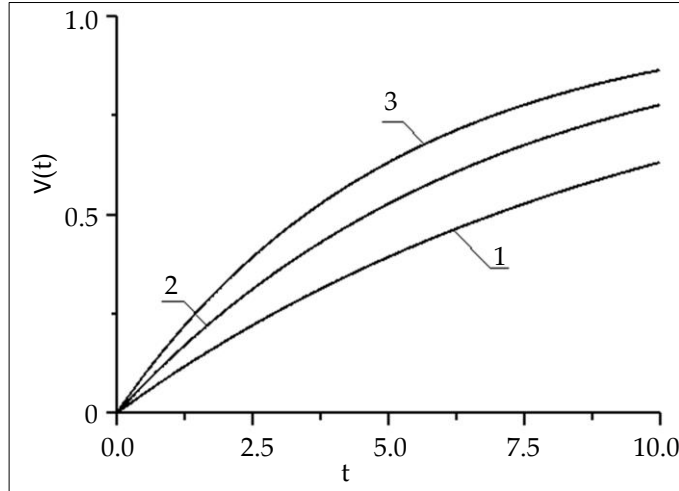


Fig. 1 Typical dependences of the volume of blood in storage tanks on time during filling of them

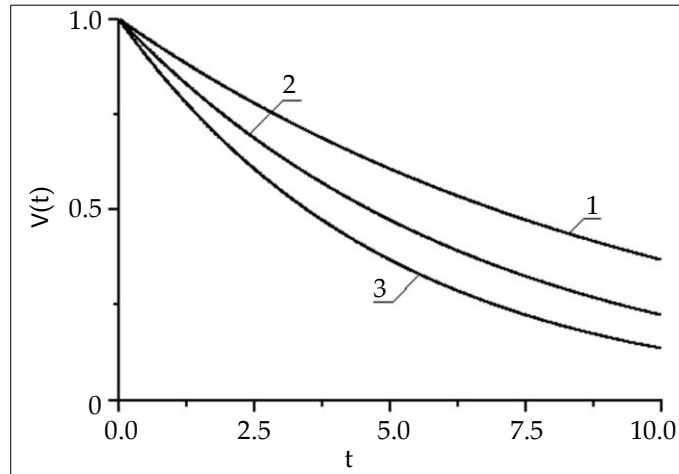


Fig. 2 Typical dependences of the volume of blood in storage tanks on time during emptying of them

### 3. Conclusion

We consider a model of blood transport during cardiac contractions. We introduce an approach for analyzing the considered model. We consider the possibility of changing the rate of blood transport.

### References

- [1] A.M. Kolsanov et al., "Virtual Surgeon Complex for Simulation Training in Surgery," *Medical Technique*, no. 6, pp. 7-10, 2013. [[Google Scholar](#)] [[Publisher Link](#)]
- [2] Kuzmin Andrey Viktorovich, "Modeling and Visualization of Cardiac Function in Computer Applications," *Bulletin of the Samara Scientific Center of the Russian Academy of Sciences*, vol. 17, no. 2-5, pp. 1031-1035, 2015. [[Google Scholar](#)] [[Publisher Link](#)]
- [3] L.I. Titomir, and P. Kneppo, *Mathematical Modeling of The Bioelectric Generator of The Heart*, Science, Fizmatlit, Russia, 1999. [[Google Scholar](#)] [[Publisher Link](#)]
- [4] A.E. Zangiev et al., "The Influence of Turbulence on The Development of Flow In a High-Speed Combustion Chamber," *Combustion and Explosion*, vol. 9, no. 3, pp. 66-79, 2016. [[Google Scholar](#)] [[Publisher Link](#)]
- [5] Y.V. Solodyannikov, *Elements of Mathematical Modeling and Identification of the Circulatory System*, Samara University Press, Samara, Russia, 1994. [[Google Scholar](#)]
- [6] Granino A. Korn, and Theresa M. Korn, *Mathematical Handbook for Scientists and Engineers. Definitions, Theorems and Formulas for Reference and Review*, McGraw-Hill Book Company, New York, USA, 1961. [[Publisher Link](#)]