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Original Article

Some Unconventional Simultaneous Cubical Equations and their Solutions

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Abstract - Some sets of simultaneous equations that have not yet found a home in the literature are innovated and are solved in this feature. That way, subtlety and insight are encountered in cubic equations.

Keywords - Cubic equations, Modelling, Mathematics, Simultaneous, Algebra.

1. Introduction

In many textbooks of Mathematics/Algebra, there is a chapter on simultaneous equations, which mostly do not encounter cubic equations in the course of obtaining their solutions. In this article several problems dealing with simultaneous equations vis-à-vis cubic equations are formed with new insight and are solved.SN Maitra[1], the present author, studied cubic equations when he encountered a cubic equation solving in which he determined the depth of the liquid poured in a hemispherical bowl from a right circular cylindrical glassful of the liquid.

2. Problem No. 1

Find integer solutions to the equations.

$$x^3 - y^3 + z^3 = 38 \quad (1)$$

$$x^2 + y^2 + z^2 = 26 \quad (2)$$

$$x+y+z=8 \quad (3)$$

2.1. Solution to Problem No. 1

If we would have y^3 instead of $-y^3$,then arises a far easier solution. In this context, We can write a well-known formula,

$$x^3 + y^3 + z^3 - 3xyz = (\mathbf{x} + \mathbf{y} + \mathbf{z})(x^2 + y^2 + z^2 - xy\text{-}yz\text{-}zx)$$

Replacing y by -y is obtained.

$$x^3 - y^3 + z^3 + 3xyz = (\mathbf{x} - \mathbf{y} + \mathbf{z})(x^2 + y^2 + z^2 + xy\text{+}yz\text{-}zx)$$

Using (1) and (2),

$$38 = (8-2y)\{(26+y(8-y)-zx\} -3xyz \quad (4)$$

Using (2) and (3),

$$x^2 + z^2 = 26 - y^2 \quad (5)$$

$$x+z=8-y \quad (6)$$

$$2xz = (8-y)^2 - (26-y^2) = 38 - 16y + 2y^2 \quad (7)$$

Using (7), (4) is rewritten as,

$$38=(8-2y)\{(26+y(8-y)-(19-8y+y^2)\} - 3y(19-8y+y^2)\}$$

$$= (8-2y)\{(7+16y-2y^2)\} - 3y(19-8y+y^2)$$

$$\text{Or, } 4y^3 - 48y^2 + 114y + 56 - 57y + 24y^2 - 3y^3 - 38 = 0$$

$$\text{Or, } y^3 - 24y^2 + 57y + 18 = 0 \quad (8)$$

By trial, y = 3 satisfies (8) and (y-3) is one factor of (8):

$$y^2(y-3) - 21y(y-3) - 6(y-3) = 0$$

$$(y^2 - 21y - 6)(y-3) = 0$$

$$Y = 3 \text{ or } y = \frac{21 \pm \sqrt{465}}{2} \quad (9)$$

Because of (9), (6) and (7) yield x+z = 5 and xz = 4.

Between which eliminating z,

$$x^2+zx = 5x$$

$$x^2 - 5x + 4 = 05x$$

$$\text{Or, } (x-4)(x-1) = 0$$

$$x = 4 \text{ or } 1 \text{ and } z = 1 \text{ or } 4$$

3. Problem No. 2

$$x^3 + y^3 + z^3 = 92 \quad (10)$$

$$x^2 + y^2 + z^2 = 26 \quad (11)$$

$$x-y+z=2 \quad (12)$$

3.1. Solution to Problem No. 2

$$x^3 - y^3 + z^3 + 3xyz = (x-y+z)(x^2 + y^2 + z^2 + xy+yz-zx) \quad (13)$$

Employing (10), (11) and (12) in (13),

$$92-2y^3+3xyz=2\{26+y(y+2)-zx\} \quad (14)$$

Combining (11) and (12),

$$2xz=(x+z)^2-(x^2+z^2)$$

$$\text{Or, } 2xz=(2+y)^2-(26-y^2)=2y^2+4y-22$$

$$\text{Or, } xz=y^2+2y-11 \quad (15)$$

Combining (12) and (15) and using (14),

$$92-2y^3+3y(y^2+2y-11)=2\{26+y(y+2)-(y^2+2y-11)\}$$

$$\text{Or, } y^3+6y^2-33y+18=0 \quad (16)$$

$$\text{Or, } y^2(y-3)+9y(y-3)-6(y-3)=0$$

$$\text{Or, } (y^2+9y-6)(y-3)=0$$

Hence,

$$y=3 \text{ or } y=\frac{-9\pm\sqrt{81+24}}{2}=\frac{-9\pm\sqrt{105}}{2} \quad (17)$$

Combining (17) with (12) and (15),

$$xz=4,\ x+z=5 \quad (18)$$

Which give eliminating z,

$$x^2-5x+4=0$$

$$\text{Or, } x^2-4x-x+4=0$$

$$\text{Or, } (x-4)(x-1)=0$$

$$x=4,1 \text{ and from (18) } z=1,4$$

Thus, we got the values of x,y and z.

4. Problem No. 3

Find the integer solutions to the equations,

$$x^3+y^3+z^3=92 \quad (19)$$

$$x^2-y^2+z^2=8 \quad (20)$$

$$x+y+z=8 \quad (21)$$

4.1. Solution to Problem No. 3

Using (20) and (21),

$$x^2 + z^2 = 8 + y^2 \quad (22)$$

$$x+z = 8-y \quad (23)$$

Combining (23) and (22),

$$2xz = (x + z)^2 - (x^2 + z^2) = (8 - y)^2 - (8 + y^2)$$

$$xz = 28-8y \quad (24)$$

Using (22) and (23) in (19) yields,

$$(x + z)^3 - 3xz(x + z) + y^3 = 92$$

$$(8 - y)^3 - 3(28 - 8y)(8 - y) + y^3 = 92$$

$$\text{Or, } 512\text{-}y^3 + 24y^2 - 192y + 192y - 672 + 84y - 24y^2 + y^3 = 92$$

$$\text{Or, } 84y = 252$$

$$y = 3 \quad (25)$$

As done earlier x = 4 or 1 and z = 1 or 4

It is interesting to note that evaluation (25) does not purport any cubic or quadratic equation.

Or otherwise (21) and (23) give,

$$x+z = 5 \quad (26)$$

$$xz = 4 \quad (27)$$

Combining (26) and (27) as earlier,

$$x\text{-}z = 1\text{or}3$$

Hence, x = 1 or 3 and z = 4 or 1 (28)

5. Problem No. 4

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{31}{30} \quad (29)$$

$$x^3 + y^3 + z^3 = 160 \quad (30)$$

$$x^2 + y^2 + z^2 = 38 \quad (31)$$

5.1. Solution to Problem No. 4

Rewriting (29),

$$xy+yz+zx = \frac{31}{30}xyz \quad (32)$$

By formula and using (29) and (33) and (32)

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) \quad (33)$$

$$160\text{-}3xyz = (x + y + z)\{((x + y + z)^2 - 3(xy + yz + zx)\} \quad (34)$$

$$160\text{-}3xyz = (x + y + z)\{((x + y + z)^2 - 3(xy + yz + zx)$$

Combining (31), (32) and (34) ,

$$(x + y + z)^2 = 38 + \frac{31}{15}xyz \quad (35)$$

$$xyz = \frac{15}{31}(t^2 - 38)$$

$$\text{Or,}160\text{-}3xyz = (x + y + z)\left(38 - \frac{31}{30}xyz \right)$$

$$= (x + y + z)\left(38 - \frac{31}{30}xyz \right)$$

Assuming,

$$x + y + z = t \quad (36)$$

and eliminating xyz in the above equation,

$$160\text{-}3xyz = t\left(38 - \frac{31}{30}xyz \right)$$

$$\text{Or, } 160\text{-}38t \text{ -}xyz(3\text{--}\frac{31}{30}t) = 0$$

$$\text{Or, } 160\text{-}38t - \frac{15}{31}(t^2 - 38)\ (3-\frac{31}{30}t) = 0$$

$$\text{Or, } \frac{1}{2}t^3 - \frac{45}{31}t^2 - 57\ t + \frac{45x38}{31}+160 = 0$$

$$\text{Or, } 31t^3 - 90t^2 - 114x31t + 90x38+320x31 = 0$$

$$\text{Or, } 31t^2(t - 10) + 220t^2 - 3534t + 90x38+320x31 = 0$$

$$\text{Or, } 31t^2(t - 10) + 220t(t - 10) + 2200t - 3534t + 3420+9920 = 0$$

$$\text{Or, } 31t^2(t - 10) + 220t(t - 10) - 1334t + 3420+9920 = 0$$

$$\text{Or, } 31t^2(t - 10) + 220t(t - 10) - 1334(t - 10) = 0$$

$$\text{Or, } 31t^2 + 220t - 1334)(t - 10) = 0$$

$$t=10 \text{ ie, } x + y + z = 10 \quad (37)$$

Using (37) in (35) is obtained.

$$xyz = 30 \tag{38}$$

Using (37) and (38) in (32) is obtained.

$$xy+yz+zx = \frac{31}{30}xyz$$

$$\frac{30}{x} + x(10 - x) = 31$$

$$\text{Or, } x^3\text{-}10x^2\text{+}31x\text{-}30 = 0 \tag{39}$$

We see that this equation is satisfied by 2.

$$\text{So, } x^2(x - 2) - 8x(x\text{-}2)\text{+}15(x\text{-}2) = 0$$

$$\text{Or, } (x^2 - 8x\text{+}15)(x\text{-}2) = 0$$

$$\text{Or, } (x\text{-}3)(x\text{-}5)(x\text{-}2) = 0 \tag{40}$$

Or, $x = 2,3,5$ or $y = 3,5,2$ or $z = 3,5,2$

6. Problem No. 5

Find integer values of x y from the equations

$$x^2 + y^2 = 25 \tag{41}$$

$$x^3 + y^3 = 91 \tag{42}$$

6.1. Solution to Problem No. 5

$$91 = (x\text{+}y)(x^2 + y^2 - xy)$$

$$91 = (x\text{+}y)(25 - xy) \tag{43}$$

$$(x + y)^2 = x^2 + y^2 + 2xy = 25\text{+}2xy$$

$$\text{Or, } xy = \frac{(x+y)^2 - 25}{2} \tag{44}$$

Using (44) in (43),

$$91 = (x\text{+}y)(25 - xy)$$

$$\text{Or, } (x\text{+}y)\{75 - (x + y)^2\} = 182 \tag{45}$$

$$\text{Assuming } x\text{+}y = t \tag{46}$$

in (45) is obtained.

$$t^3 - 75t + 182 = 0$$

Or, $t^2(t-7)+7\,t(t-7)-26(t-7)=0$

Or, $(\,t^2+7\,t-26)(t-7)=0$

$t = 7$ or $t = \frac{-7\pm\sqrt{153}}{2}$, (47)

Using (47) and (46) in (44),

$xy = 12$, $x+y = 7$

Which, as done earlier, led to

$x = 3$,$y = 4$ or $x = 4$,$y = 3$

References

[1] SN Maitra, "A Novel Application of Cubic Equation," *International Journal of Mathematics and Statistics Innnovation*, vol. 9, no. 3, pp. 1-8, 2021. [CrossRef] [Publisher Issue]