

Original Article

Transforming Noisy Images Wavelet-Based Denoising Methods

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Received: 27 September 2023; Revised: 03 October 2023; Accepted: 14 November 2023; Published: 24 November 2023

Abstract - This paper presents a novel thresholding function that mixes the soft thresholding functions and Smoothly Clipped Absolute Deviation for denoising images using the decimated wavelet transform technique, which is widely popular in various applications. The proposed method is applied to denoise noisy images contaminated with additive white Gaussian noise, employing the Top rule method. The efficiency of this new thresholding function is also evaluated within the context of the Translation Invariant method. The outcomes are associated with standing methods such as SCAD, soft thresholding, and the Wiener filter-based denoising approach. Parameters such as root mean square error and peak signal to noise ratio are employed to assess the quality of denoising.

Keywords - WT, DWT, Image denoising, New thresholding function, Top rule, Wiener filter, Translation invariant method.

1. Introduction

Two-dimensional signals are developed in various approaches, such as digital television communication and medical analysis, and they often need to be conveyed beginning one location to another. Through this transmission method, noise can be introduced into the images, degrading their clarity and quality. Image-denoising techniques are essential to recover the original image and reduce the impact of noise. One widely used approach for this purpose is the application of wavelet transform, renowned for its excellent localization properties, in the area of one-dimensional and two-dimensional processing. Wavelet Transform (WT) denoising aims to remove noise from a one-dimensional or two-dimensional though preserving its essential features, irrespective of its frequency content. Among the dissimilar approaches available for image denoising, those based on wavelet transform are particularly popular. This project proposes a denoising method that relies on wavelet transform and employs a thresholding technique.

2. Image Denoising

In the area of two-dimensional processing, a significant challenge arises when dealing with noisy images. Image denoising is the process of recovering the actual two-dimensional signal after a noisy experimental two-dimensional. The wavelet shrinkage technique used for two-dimensional denoising includes the subsequent steps:

- First, apply wavelet decomposition to the noisy image to obtain the wavelet coefficients: $p=P(B+N)$. Here, p denotes wavelet quantities, P represents the WT, N represents the noise information, and B represents the real two-dimensional signal.



- Next, put on the thresholding function via predefined thresholding instructions to the wavelet coefficients: $P' = \delta \Sigma(p)$ from the equation, P' denotes the optimal valuation of the wavelet numbers, $\delta \Sigma(p)$ denotes the wavelet thresholding function, and Σ is the threshold.
- Finally, go through the reverse WT to the changed wavelet coefficient to get the denoised two-dimensional signal: $B' = P^{-1}(P')$.

3. Shrinkage Methods

3.1. Top Rule

This method is a global shrinkage function that remains independent of the selected shrinkage function. Let us consider "K" by means of the section of the major coefficients we wish to retain. The shrinkage stands set near be located the (1-K)th quintile of the experimentally derived complete values of the wavelet coefficients.

3.2. Translation Invariant Method

Donoho and Coifman announced the translational invariant method involving two key steps. First, it entails performing shrinkage on each basis, and second, it averages the resulting denoised signals. This approach is effective for two main reasons: Enhanced Detection of Singularities: By accounting for all shifts in the analysis, it improves the detection of singularities in the data. Improved Smoothing Effect: A more powerful smoothing effect is achieved by averaging the denoised signals obtained from each basis. To implement this strategy effectively, unbalanced wavelets with small support are employed, and soft shrinkage is applied. This method not only enhances edge compensation but is also commonly used for image estimation.

4. Thresholding Functions

4.1. Soft Thresholding Function

If $p > \lambda$, then the value is $p - \lambda$. Otherwise, the value is zero.

$$S(p, \lambda) = \begin{cases} \text{sgn}(p)(|p| - \lambda) & \text{for } |p| > \lambda \\ 0 & \text{otherwise} \end{cases}$$

4.2. Smoothly Clipped Absolute Deviation

$$\text{SCAD}(p, \lambda) = \begin{cases} \text{sign}(p) \max(0, |p| - \lambda) & \text{if } |p| \leq 2\lambda \\ (\alpha - 1)p - \alpha \lambda \text{sign}(p) & \text{if } 2\lambda \leq |p| < \alpha \lambda \\ p & \text{if } |p| > \alpha \lambda \end{cases}$$

4.3. Novel Shrinkage Function

This method is developed by combining elements of the smoothly clipped absolute deviation function and the soft thresholding function, specifically through an arithmetic mean. The function's behavior is as follows: If $p > \lambda$ outputs the value as is. However, if the coefficient value is a smaller amount than the threshold, the function computes the output as 20% of the coefficient value.

4.4. Wiener Filter

The Wiener filter can be employed in image processing to eliminate noise from an image. Wiener filtering is a linear estimation method for denoising. It serves to minimize the total mean square error during the procedure of inverse filtering and noise smoothing. The following equations show the winner filter.

$$W(y, z) = \frac{H^*(y, z) S_{xx}(y, z)}{|H(y, z)|^2 S_{xx}(y, z) + S_{\eta\eta}(y, z)}$$

Where $S_{\eta\eta}(y, z)$ $S_{xx}(y, z)$ are the additive noise and power spectral of the original two-dimensional signal, and $H(y, z)$ is the blurring function.

5. Results

In this section, we present the outcomes of our experiments and engage in a comprehensive discussion. We calculated the effectiveness of the proposed thresholding function and compared it to both the soft and SCAD thresholding functions, all implemented using wavelet; the experiment is carried out on a 512X512 image with various levels of Gaussian noise added to the original.

Our methodology involves the application of wavelet transforms to acquire the wavelet coefficients, the modification of these coefficients in accordance with the chosen shrinkage function, and the subsequent utilization of inverse wavelet transforms to generate either the denoised or original image.

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (X(i) - \hat{X}(i))^2} \quad (1)$$

$$\text{PSNR} = 20 \log_{10} (255/\sqrt{MSE})$$

Here, $X(i)$ is the actual two-dimensional signal, $\hat{X}(i)$ is the noise-cancelling two-dimensional signal, and n is the quantity of samples. The reproduction trial was frequent a hundred times, and the middling principles were computed. The process was conducted on dissimilar images, yielding consistent results. The simulation was implemented within the MATLAB environment.

5.1. Results Tables

Tables 1 and 2 display the results of two-dimensional signals for noise levels standard deviations =10, 20, and 30 using Soft, Smoothly Clipped Absolute Deviation and a novel shrinkage function with translation invariant and top rule methods. Meanwhile, Table 3 presents the denoising results using the Wiener filter method.

5.2. Visual Representation

Figures 1-4 showcase the actual and denoised two-dimensional signal using the new thresholding function with the top rule and translation invariant methods. Additionally, Figure 5 displays the denoised image using the Wiener filter method.

5.3. Comparison Graphs

Figures 6-11 illustrate the comparative analysis of results obtained from the translation invariant, top rule, and Wiener filter methods.

5.4. Quantitative Results

For standard deviation 10, when using the SCAD filter, we obtained an RMSE of 9.1899 and a PSNR of 28.8645 for denoising the noisy image. In contrast, using soft thresholding with the translation invariant method, we obtained an RMSE of 9.5555 and a PSNR of 28.5257 (Table 1).

Notably, the new thresholding function yielded an RMSE of 7.6071 and a PSNR of 30.5064, demonstrating superior performance compared to both SCAD and soft thresholding functions. Similar favourable results were observed for standard deviations for 20 and 30, as shown in Table 1.

5.5. Top Rule Method

For standard deviation 10 using the top rule method, we obtained an RMSE of 7.9138 and a PSNR of 30.1631 with the SCAD thresholding function while using the soft thresholding filter resulted in an RMSE of 6.8959 and a PSNR of 31.3590. Meanwhile, the Wiener method yielded an RMSE of 5.7431 and an SNR of 32.9479 (Table 3). The new thresholding function exhibited an RMSE of 6.6623 and a PSNR of 31.6583 (Table 2). These results consistently highlight the superior performance of the novel shrinkage function compared to both smoothly

clipped absolute deviation and soft shrinkage functions for standard deviations 10, 20, and 30. In conclusion, the novel shrinkage function consistently outperforms SCAD and soft thresholding functions across different noise levels.

Table 1. Denoising results of Lena's image by using the translation invariant method

Standard Deviation	10		20		30	
Parameters	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR
Noisy Image	10.030	28.1042	19.9966	22.1117	30.1327	18.5500
SCAD Function	9.1899	28.8645	13.8675	25.2908	17.8197	23.1128
Soft Function	9.5555	28.5257	14.0683	25.1660	17.6287	23.2064
New Thresholding Function	7.6071	30.5064	11.1514	27.1842	14.1289	25.1286

Table 2. Denoising results of Lena's image by using the top rule

Standard Deviation	10		20		30	
Parameters	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR
Noisy Image	9.9900	28.1395	19.9712	22.1227	30.1073	18.5574
SCAD Function	7.9138	30.1631	12.7692	26.0075	19.3086	22.4158
Soft Function	6.8959	31.3590	11.1603	27.1773	14.3746	24.9789
New Thresholding Function	6.6623	31.6583	11.4799	26.9320	17.4095	23.3151

Table 3. Denoising results of Lena's image by using the Wiener filter method

Standard Deviation	10		20		30	
Parameters	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR
Noisy Image	9.9653	28.1610	19.9630	22.1263	29.8474	18.6327
Wiener Filter	5.7431	32.9479	8.8244	29.2171	10.9144	27.3708



Fig. 1 Original image



Fig. 2 Noisy image ($\sigma=30$)



Fig. 3 Denoised image using novel shrinkage with translation invariant method



Fig. 4 Denoised image using novel shrinkage function with top rule



Fig. 5 Denoised image using Wiener filter

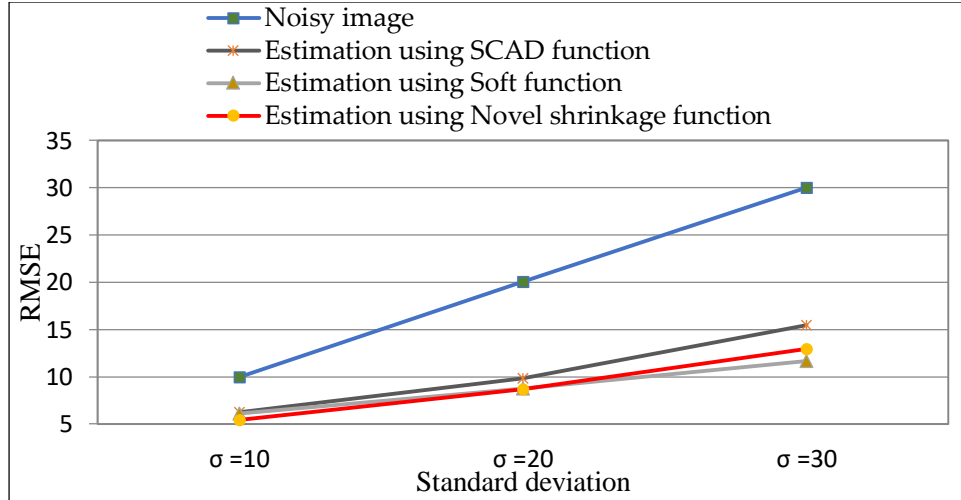


Fig. 6 Translation invariant method for different values of RMSE

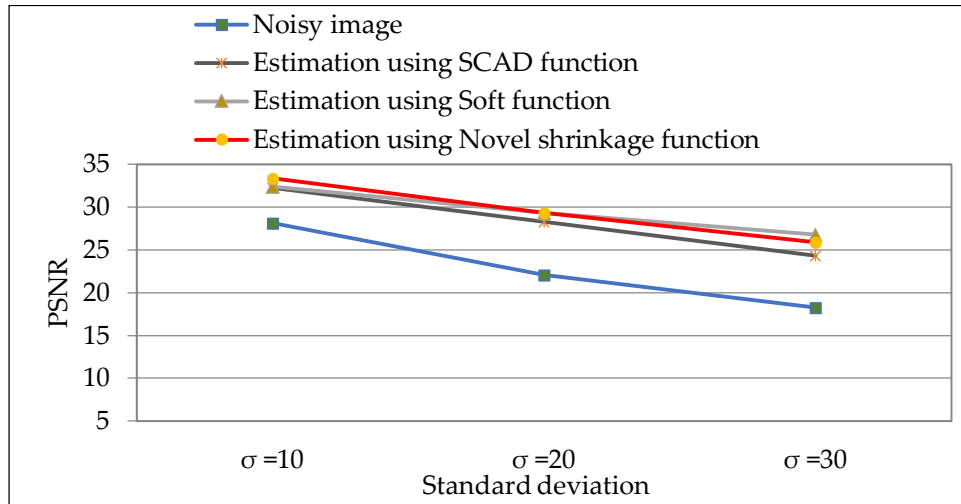


Fig. 7 Translation invariant method for different values of PSNR

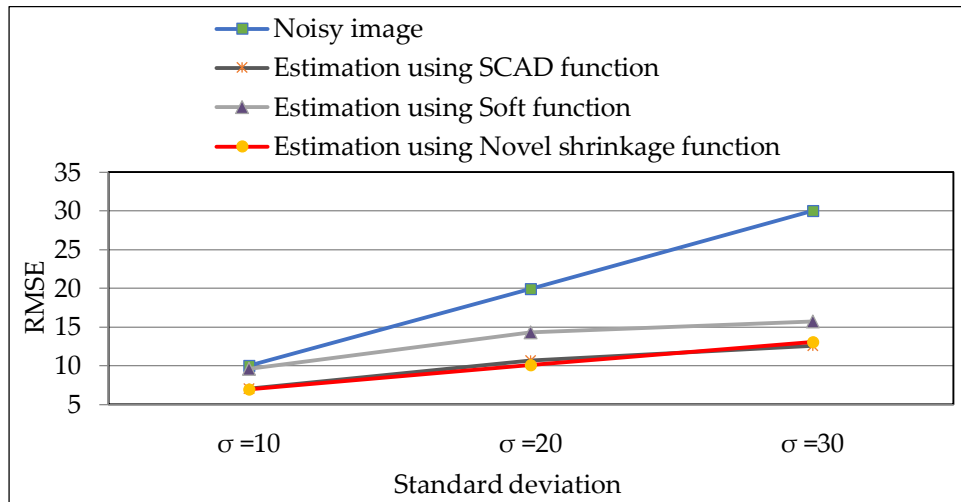


Fig. 8 Top rule method for different values of RMSE

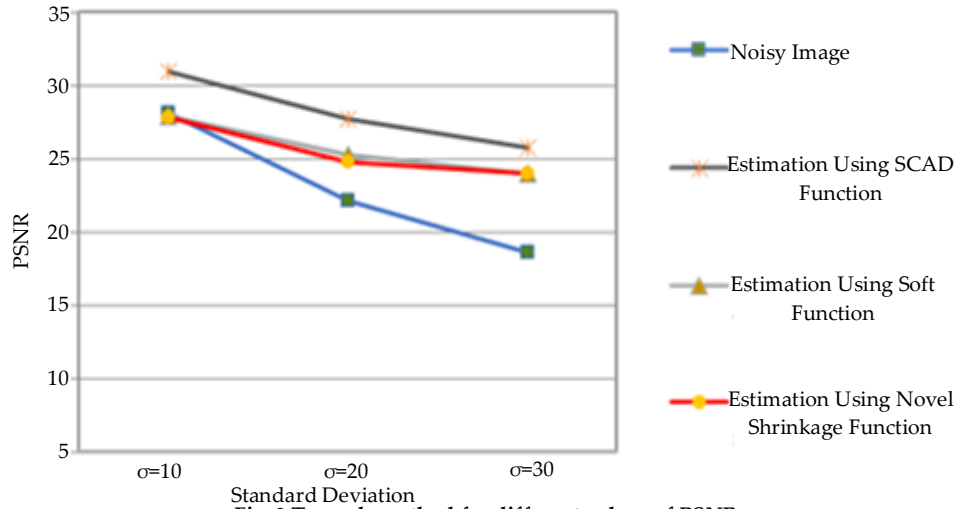


Fig. 9 Top rule method for different values of PSNR

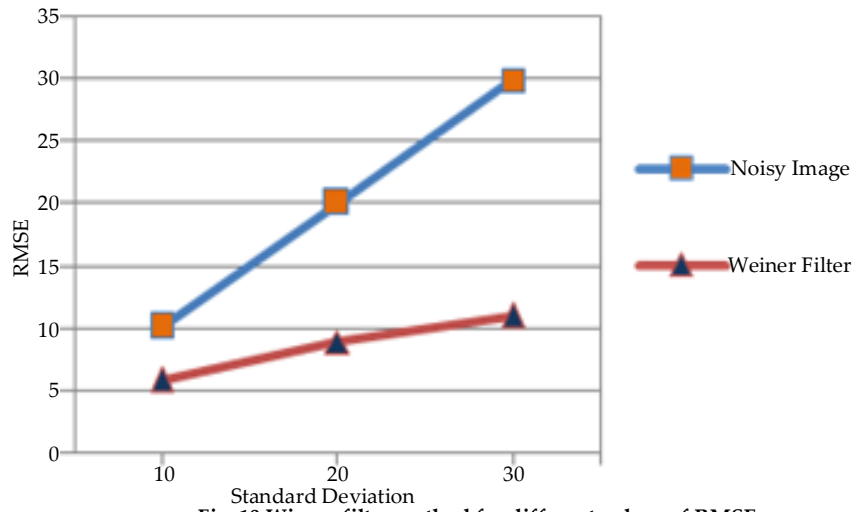


Fig. 10 Wiener filter method for different values of RMSE

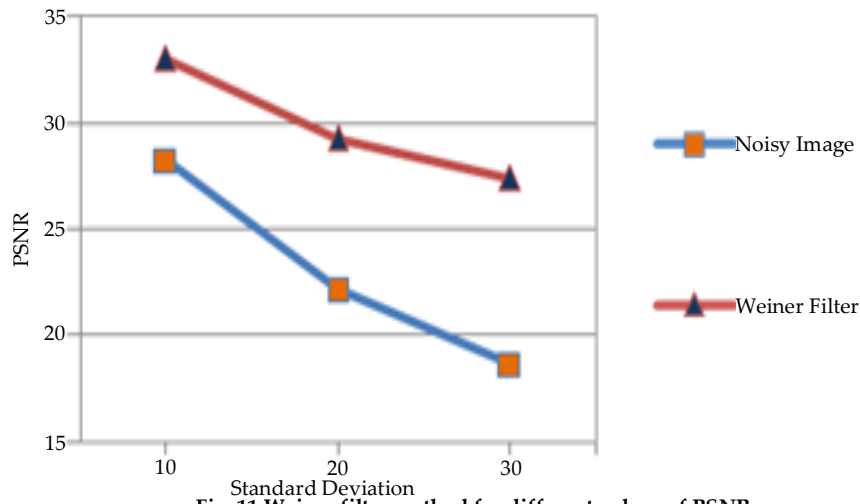


Fig. 11 Wiener filter method for different values of PSNR

6. Conclusion

In this paper, a novel shrinkage function is proposed for wavelet shrinkage denoising of two-dimensional signals; the exhibition of this function is estimated using the Lena image and related to present SCAD, soft, and Wiener filter methods. At this time, we can see that the novel shrinkage function gives improved results than the existing methods; both SCAD and soft functions with the Translation invariant method outperform the SCAD function with the Top rule and outperform the existing Wiener filter method.

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